Transfer of a Polaritonic Qubit through a Coupled Cavity Array

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Abstract

We demonstrate a scheme for quantum communication between the ends of an array of coupled cavities. Each cavity is doped with a single two level system (atoms or quantum dots) and the detuning of the atomic level spacing and photonic frequency is appropriately tuned to achieve photon blockade in the array. We show that in such a regime, the array can simulate a dual rail quantum state transfer protocol where the arrival of quantum information at the receiving cavity is heralded through a fluorescence measurement. Communication is also possible between any pair of cavities of a network of connected cavities.

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I. INTRODUCTION

Recently, the exciting possibility of coupling high Q cavities directly with each other has materialized in a variety of settings, namely fiber coupled micro-toroidal cavities [1], arrays of defects in photonic band gap materials (PBGs) [2, 3] and microwave stripline resonators joined to each other [4]. A further exciting development has been the ability to couple each such cavity to a quantum two-level system which could be atoms for micro-toroid cavities, quantum dots for defects in PBGs or superconducting qubits for microwave stripline resonators[5]. Possibilities with such systems are enormous and include the the implementation optical quantum computing [6], the production of entangled photons [7], the realization of Mott insulating and superfluid phases and spin chain systems [8, 9, 10] . Such settings can also be used to verify the possibilities of distributed quantum computation involving atoms coupled to distinct cavities [11] also to generate cluster states for efficient measurement based quantum computing schemes[12].

When the coupling between the cavity field and the two-level system (which we will just call atom henceforth, noting that they need not necessarily be only atoms) is very strong (in the so called strong coupling regime), each cavity-atom unit behaves as a quantum system whose excitations are combined atom-field excitations called polaritons. The nonlinearity induced by this coupling or as it is otherwise known, the photon blockade effect[13], forces the system to a state where maximum one excitation (polariton) per site is allowed. However, a superposition of two different polaritons, which is equivalent to a superposition of two energy levels of the cavity-atom system, is indeed allowed and naturally the question arises as to whether that can be used as a qubit. Purely atomic qubits (formed from purely atomic energy levels) in cavities have long been discussed in the literature (see references cited in [11], for example), but such qubits in distinct cavities do not directly interact with each other unless mediated through light. On the other hand, a purely photonic field in a cavity is not easy to manipulate in the sense of one being able to create arbitrary superpositions of its states by an external laser. Being a mixed excitation, polaritons interact with each other as well as permit easy manipulations with external lasers in much the same manner as one would manipulate and superpose atomic energy levels. Is there any interesting form of quantum information processing that can be performed by encoding the quantum information in a superposition of polaritonic states? While an ultimate aim might be to accomplish full quantum computation

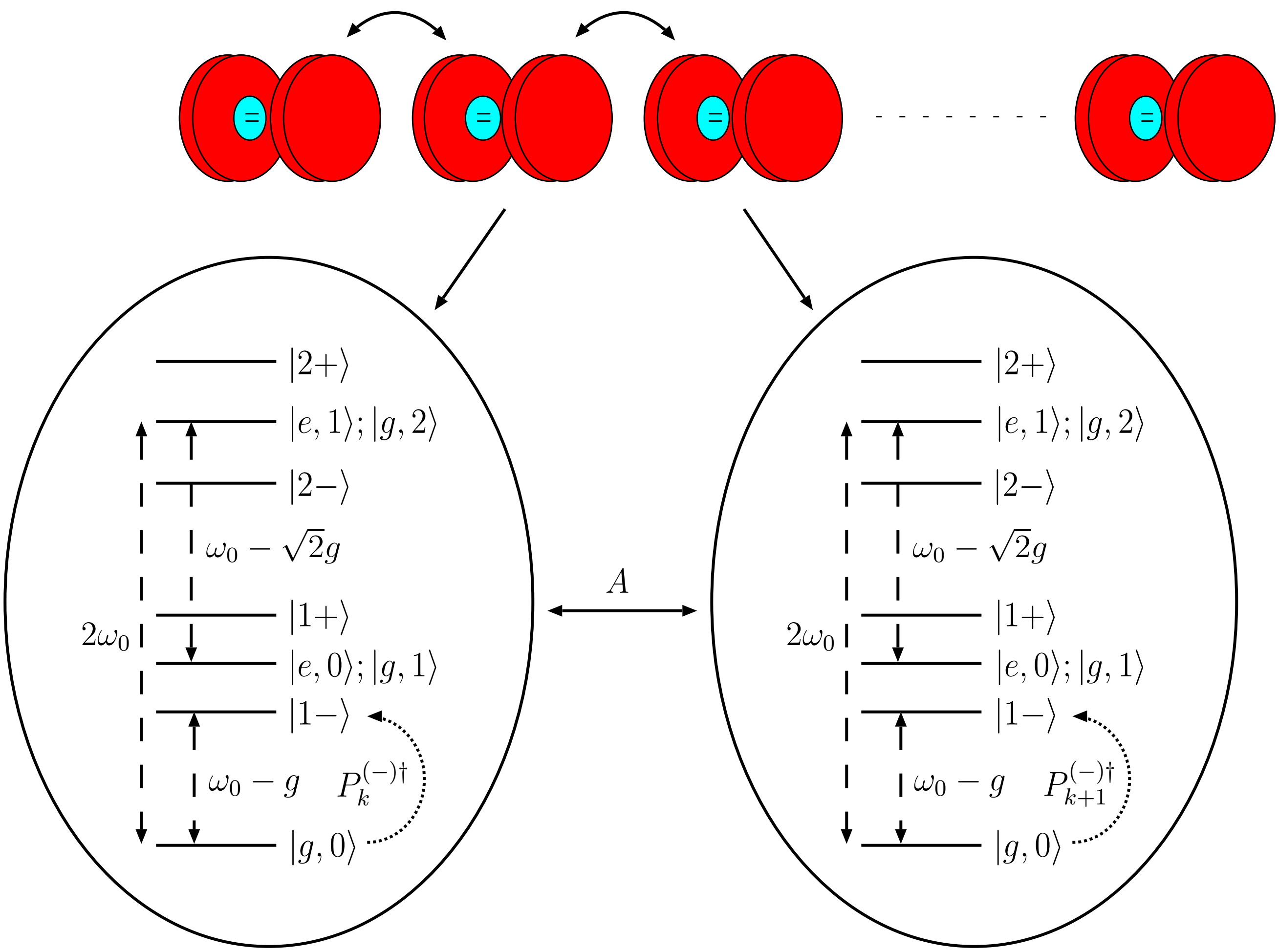

FIG. 1: A series of coupled cavities coupled through light and the polaritonic energy levels for two neighbouring cavities. These polaritons involve an equal mixture of photonic and atomic excitations and are defined by creation operators $P_k^{(\pm,n)\dagger} = (|g,n\rangle_k\langle g,0|_k \pm |e,n-1\rangle_k\langle g,0|_k)/\sqrt{2}$, where $|n\rangle_k, |n-1\rangle_k$ and $|0\rangle_k$ denote $n, n-1$ and 0 photon Fock states in the kth cavity. The polaritons of the kth atom-cavity system are denoted as $|n\pm\rangle_k$ and given by $|n\pm\rangle_k = (|g,n\rangle_k \pm |e,n-1\rangle_k)/\sqrt{2}$ with energies $E_n^{\pm} = n\omega_d \pm g\sqrt{n}$.

with polaritonic qubits (it has been recently shown this to possible using the cluster state approach [12]), we concentrate here on a more modest aim of transferring the state of a qubit encoded in polaritonic states (a polaritonic qubit) from one end of the coupled cavity array to another.

Assume a chain of N coupled cavities. We will describe the system dynamics using the operators corresponding to the localized eigenmodes (Wannier functions), $a_k^{\dagger}(a_k)$. The

Hamiltonian is given by

$$H = \sum_{k=1}^{N} \omega_d a_k^\dagger a_k + \sum_{k=1}^{N} A(a_k^\dagger a_{k+1} + H.C.). \tag{1}$$

and corresponds to a series quantum harmonic oscillators coupled through hopping photons. The photon frequency and hopping rate is ω_d and A respectively and no nonlinearity is present yet. Assume now that the cavities are doped with two level systems (atoms/ quantum dots/superconducting qubits) and $|g\rangle_k$ and $|e\rangle_k$ their ground and excited states at site k. The Hamiltonian describing the system is the sum of three terms. H^{free} the Hamiltonian for the free light and dopant parts, H^{int} the Hamiltonian describing the internal coupling of the photon and dopant in a specific cavity and H^{hop} for the light hopping between cavities.

$$\begin{aligned} H^{free} &= \omega_d \sum_{k=1}^{N} a_k^\dagger a_k + \omega_0 \sum_k |e\rangle_k \langle e|_k &(2)\\ H^{int} &= g \sum_{k=1}^{N} (a_k^\dagger |g\rangle_k \langle e|_k + H.C.) &(3)\\ H^{hop} &= A \sum_{k=1}^{N} (a_k^\dagger a_{k+1} + H.C) &(4) \end{aligned}$$

where g is the light atom coupling strength. The $H^{free} + H^{int}$ part of the Hamiltonian can be diagonalized in a basis of mixed photonic and atomic excitations, called *polaritons* (Fig. 1). While $|g,0\rangle_k$ is the ground state of each atom cavity system, the excited eigenstates of the kth cavity-atom system are given by $|n\pm\rangle_k = (|g,n\rangle_k \pm |e,n-1\rangle_k)/\sqrt{2}$ with energies $E_n^\pm = n\omega_d \pm g\sqrt{n}$. One can then define polariton creation operators $P_k^{(\pm,n)\dagger}$ by the action $P_k^{(\pm,n)\dagger}|g,0\rangle_k = |n\pm\rangle_k$. As we have proved elsewhere, due to the blockade effect, once a site is excited to $|1-\rangle$ or $|1+\rangle$, no further excitation is possible[8]. In simplified terms, this is because it costs more energy to add another excitation in already filled site so the system prefers to deposit it if possible to an a nearby empty site. This effect has recently lead to the prediction of a Mott phase for polaritons in coupled cavity systems[8]. If we restrict to the low energy dynamics of the system such that states with $n \geq 1$ are not occupied, which can be ensured through appropriate initial conditions, the Hamiltonian in becomes (in the interaction picture):

$$H_I = A \sum_{k=1}^{N} P_k^{(-)\dagger} P_{k+1}^{(-)} + A \sum_{k=1}^{N} P_k^{(+)\dagger} P_{k+1}^{(+)} + H.C. \tag{5}$$

where $P_k^{(\pm)\dagger} = P_k^{(\pm,1)\dagger}$ is the polaritonic operator creating excitations to the first polaritonic manifold (Fig. 1). In deriving the above, the logic is that the terms of the type $P_k^{(-)\dagger} P_{k+1}^{(+)}$, which inter-convert between polaritons, are fast rotating and they vanish[8].

We are now in a position to outline the basic idea behind the protocol. A qubit is encoded as a superposition of the polaritonic states $|1+\rangle$ and $|1-\rangle$ in the first cavity. The multi-cavity system is then allowed to evolve according to H_I. At the receiving cavity at the other end we then do a measurement inspired by a dual rail quantum state transfer protocol [14] which heralds the perfect reception of the qubit for one outcome of the measurement, while for the other outcome of the measurement the process is simply to be repeated once more after a time delay. Before presenting the scheme in detail, let us first present a special set of initial conditions under which H_I describes the dynamics of two identical parallel uncoupled spin chains.

Suppose we are restricting our attention to a dynamics in which the initial state is obtained by the action of only one of the operators among $P_k^{(+)\dagger}$ and $P_k^{(-)\dagger}$ on the state $\prod_k |g,0\rangle_k$ which has all the sites in the state $|g,0\rangle$. As $P_k^{(-)\dagger}$ does not act after $P_k^{(+)\dagger}$ has acted and vice versa, under the above restricted initial conditions, the system is going to evolve only according to one of the terms in Eq.(5) *i.e.,* only according to the first or the second term. To be more precise, if we start with a state $P_j^{(+)\dagger} \prod_k |g,0\rangle_k$ only the term $A\sum_{k=1}^{N} P_k^{(+)\dagger} P_{k+1}^{(+)}$ is going to be active and cause the time evolution, while if we start with the state $P_j^{(-)\dagger} \prod_k |g,0\rangle_k$ only the term $A\sum_{k=1}^{N} P_k^{(-)\dagger} P_{k+1}^{(-)}$ will be responsible for the time evolution. Each of the operators $P_k^{(+)\dagger}$ and $P_k^{(-)\dagger}$ individually have the same algebra as the Pauli operator $\sigma_k^+ = \sigma_k^x + i\sigma_k^y$, which makes both the parts of the Hamiltonian individually equivalent to a XY spin chain with a Hamiltonian $H_{XY} = A\sum_k(\sigma_k^x\sigma_{k+1}^x + \sigma_k^y\sigma_{k+1}^y)$. The restricted set of initial states mentioned above can be mapped on to those of two parallel chains of spins labeled as chain I and chain II respectively. Let $|0\rangle$ and $|1\rangle$ be spin-up and spin-down states of a spin along the z direction, $|\mathbf{0}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}$ be a state with all spins of both chains being in the state $|0\rangle$, $|\mathbf{k}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}$ represent the state obtained from $|\mathbf{0}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}$ by flipping only the kth spin of chain I and $|\mathbf{0}\rangle^{(I)}|\mathbf{k}\rangle^{(II)}$ represents the state obtained from $|\mathbf{0}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}$ by flipping only the kth spin of chain II. Then, the restricted class of initial conditions for polaritonic states can be mapped on to states of the parallel spin chains as

$$|g,0\rangle_1|g,0\rangle_2....|g,0\rangle_N \rightarrow |\mathbf{0}\rangle^I|\mathbf{0}\rangle^{II}, \tag{6}$$

$$|g,0\rangle_1..|g,0\rangle_{k-1}|1+\rangle_k|g,0\rangle_{k+1}..|g,0\rangle_N \rightarrow |\mathbf{k}\rangle^{(I)}|\mathbf{0}\rangle^{II}, \tag{7}$$

$$|g,0\rangle_1..|g,0\rangle_{k-1}|1-\rangle_k|g,0\rangle_{k+1}..|g,0\rangle_N \rightarrow |\mathbf{0}\rangle^{I}|\mathbf{k}\rangle^{(II)} \tag{8}$$

Under the above mapping and under the above restrictions on state space, H_I becomes equivalent to the Hamiltonian of two identical parallel XY spin chains completely decoupled from each other. Precisely such a Hamiltonian is known to permit a heralded perfect quantum state transfer from one end of a pair of parallel spin chains to the other [14], and we discuss that below.

Spin chains are capable to transmitting quantum states by natural time evolution [15]. However it is well known that due to the disperion on the chain [16] the fidelity of transfer is quite low except for specific engineered couplings in the spin chains [17, 18] or when the receiver has access to a significant memory [19]. The advantage of the polariton system is that we have *two parallel and identical* chains. We have recently shown how this can be made use of in a dual rail protocol [14]. The main idea of this protocol is to encode the state in a symmetric way on both chains. The sender Alice encodes a qubit $\alpha|0\rangle + \beta|1\rangle$ to be transmitted as

$$|\Phi(0)\rangle = \alpha|\mathbf{0}\rangle^{(I)}|\mathbf{1}\rangle^{(II)} + \beta|\mathbf{1}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}, \tag{9}$$

which evolves with time as

$$|\Phi(t)\rangle = \sum_{j=1}^{N} f_{1j}(t)(\alpha|\mathbf{0}\rangle^{(I)}|\mathbf{j}\rangle^{(II)} + \beta|\mathbf{j}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}), \tag{10}$$

where f_{1j} is the transition amplitude of a spin flip from the 1st to the jth site of a chain. Clearly, if after waiting a while Bob performs a joint parity measurement on the two spins at his (receiving) end of the chain and the parity is found to be "odd", then the state of the whole system will be projected to $\alpha|\mathbf{0}\rangle^{(I)}|\mathbf{N}\rangle^{(II)} + \beta|\mathbf{N}\rangle^{(I)}|\mathbf{0}\rangle^{(II)}$, which implies the perfect reception of Alice's state (albeit encoded in two qubits now). The protocol presented in Ref.[14] in fact suggested the use of a two qubit quantum gate at Bob's end which measured both the parity as well as mapped the state to a single qubit state. However, here the presentation as above suffices for what follows. Physically, this protocol, which is called the dual rail protocol, allows one to perform measurements on the chain that monitor the location of the quantum information *without perturbing it*. As such it can also be used for arbitrary graphs of spins (as long as there are two identical parallel graphs) with the receiver at any node of the graph. Furthermore, for the Hamiltonian at hand (XY spin model) it is

known [20] that the probability of success converges exponentially fast to one if the receiver performs regular measurements. The time it takes to reach a transfer fidelity F scales as

$$t = 0.33A^{-1}N^{5/3}|\ln(1-F)|. \tag{11}$$

The difference between our current coupled cavity system and the spin chain system considered in [14] is that in our case, the two chains are effectively realized in *one* system. Therefore, it is not necessary to perform a two-qubit measurement such as a parity measurement at the receiving ends of the chain. The qubit to be transferred is encoded as $\alpha^{'}|1+\rangle_1 + \beta^{'}|1-\rangle_1 \equiv \alpha|e,0\rangle_1 + \beta|g,1\rangle_1$. This state can be created by the sender Alice using a resonant Jaynes-Cummings interaction between the atom and the cavity field. Then the whole evolution will exactly be as in Eq.(10) with the spin chain states have to be replaced by polaritonic states according to the mapping given in Eqs.(6)-(8). The measurement to herald the arrival of the state at the receiving end is accomplished by a exciting (shelving) $|g,0\rangle$ repeatedly to a metastable state by an appropriate laser (which does not do anything if the atom is either in $|1\pm\rangle$). The fluorescence emitted on decay of the atom from this metastable state to $|g,0\rangle$ implies that another measurement has to be done after waiting a while. No fluorescence implies success and completion of the perfect transfer of the polaritonic qubit. Interestingly enough, the measurement at the receiving cavity need not be snapshot measurements at regular time intervals, but can also be continuous measurements under which the scheme can have very similar behavior to the case with snap-shot measurements for appropriate strength of the continuous measurement process [21].

We now briefly discuss the parameter regime needed for the scheme of this paper. In order to achieve the required limit of no more than one excitation per site, the parameters should have the following values[8]. The ratio between the internal atom-photon coupling and the hopping of photons down the chain should be $g/A = 10^2$. We should be on resonance, $\Delta = 0$, and the cavity/atomic frequencies $\omega_d, \omega_0 \sim 10^4 g$ which means we should be well in the strong coupling regime. The losses should also be small, $g/max(\kappa, \gamma) \sim 10^3$, where κ and γ are cavity and atom/other qubit decay rates. These values are expected to be feasible in both toroidal microcavity systems with atoms and stripline microwave resonators coupled to superconducting qubits [5], so that the above states are essentially unaffected by decay for a time $10/A$ (10ns for the toroidal case and 100ns for microwave stripline resonators type of implementations).

We conclude with a brief discussion about the positive features of the scheme and situations in which the scheme might be practically relevant. The scheme combines the best aspects of both atomic and photonic qubits as far as communication is concerned. The atomic content of the polaritonic state enables the manipulation to create the initial state and measure the received state of the cavity-atom systems with external laser fields, while the photonic component enables its hopping from cavity to cavity thereby enabling transfer. Unlike quantum communication schemes where an atomic qubit first has to be mapped to the photonic state in the transmitting cavity and be mapped back to an atomic state in the receiving cavity by external lasers, here the polaritonic qubit simply has to be created. Once created, it will hop by itself though the array of cavities without the need of further external control or manipulation.

In what situations might such a scheme have some practical utility? One case is when Alice "knows" the quantum state she has to transmit to Bob. She can easily prepare it as a polaritonic state in her cavity and then let Bob receive it through the natural hopping of the polaritons. Another situation is when a multiple number of cavities are connected with each other through an arbitrary graph. The protocol of Ref.[14] still works fine in this situation with Alice's qubit being receivable in any of the cavities simply by doing the receiving fluorescence measurements in that cavity.

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